Int. J. Heat Mass Transfer Vol. 18, pp 553–557 Pergamon Press 1975. Printed in Great Britain

UNIFORM HEAT FLUX IN A PAPER DRYING DRUM WITH A NON-CYLINDRICAL CONDENSATION SURFACE OPERATING UNDER RIMMING CONDITIONS

WILFRIED ROETZEL* and MICHAEL NEWMAN†
Chemical Engineering Research Group, Council for Scientific and Industrial Research, Pretoria, South Africa

(*Received* 20 *March* 1974)

Abstract—In a paper dryer a uniform heat flux along the surface of the drum is desirable. To obtain this a wall profile is developed which yields a uniform overall heat-transfer resistance of wall and condensate film.

Using an incremental calculation it is shown that the assumptions made are valid and that the developed profile is suitable.

An equation is presented for estimating the mean heat-transfer coefficient for condensation with the developed wall profile.

NOMENCLATURE

A, dimensionless group defined by (3);
a, exponent;
b, acceleration;
G, dimensionless group defined by (18);
h, latent heat of condensation;
K, constant;
L, total flow length of condensate;
Nu, mean Nusselt number for constant heat flux;
$\dot{q}$, uniform heat flux per unit time and area;
r, mean inside radius of drum;
s, substitution variable defined by (21);
t, substitution variable defined by (22);
T, temperature;
w, wall thickness;
x, variable flow length of condensate film;
y, total thickness of condensate film.

Greek symbols

δ, dimensionless wall thickness defined by (7);
λ, thermal conductivity;
ν, mean kinematic viscosity of condensate;
ξ, dimensionless flow length of condensate film defined by (2);
ρ, density of condensate;
σ, total equivalent dimensionless wall thickness representing the total heat-transfer resistance of condensate film and wall defined by (8);
ϕ, dimensionless thickness of condensate film defined by (1);
ω, angular velocity of rotating system.

Subscripts

1, at the point where $\xi = 1$;
c, of the condensate;
d, design conditions;
i, counter for incremental calculation;
n, normal to the wall;
w, of the wall;
x, in the direction of x.

*Present address: Bayer AG, 415 Krefeld, F.R. Germany.
†Present address: Abu Dhabi Petroleum Company Ltd., Abu Dhabi, Arabian Gulf.

1. INTRODUCTION

RECENTLY it was shown [1] that the rate of heat transfer inside fast rotating paper drying drums can be increased considerably by adopting a construction which differs from the conventional one [2] in that it uses a drum with a curved conical inside surface. In [1] the special case was treated where the inside surface of the drum has a circular curvature in the axial direction. This results in a condensate film of nearly uniform thickness, which gives a nearly uniform overall heat-transfer resistance for cases where the drum inside surface has a large radius of curvature and the wall material has a high thermal conductivity. Strictly, the overall resistance is uniform for this construction only when the ratio of wall thermal conductivity to condensate thermal conductivity is infinite.

This investigation is concerned with trying to obtain a construction which gives a more uniform overall resistance of condensate film and tube wall for any value of the wall thermal conductivity.

For convenience the nomenclature has been taken to be the same as in [1] wherever possible.

2. DEVELOPMENT OF EQUATIONS

As in [1] we define the dimensionless film thickness and flow path

$$\phi = \frac{y}{L} \tag{1}$$

$$\xi = \frac{x}{L} \tag{2}$$

and the dimensionless group

$$A = \frac{b_n . \rho . h . L^2}{6 . \nu . \dot{q}} \tag{3}$$

where the normal forces b_n are given by

$$b_n = \omega^2 . r. \tag{4}$$

The relative change of the drum radius in the axial direction is very small and so r in (4) can be taken as a mean value. The condensate film flow can be described by

$$\frac{d\phi}{d\xi} = 2 . \frac{b_x}{b_n} - \frac{1}{A} . \frac{\xi}{\phi^3} \tag{5}$$

as shown previously [1]. The differential equation (5) is valid for uniform heat flux. It was derived with the assumption that a Nusselt [3] velocity profile exists in the condensate film. As in [1] we assume that the drum inside surface is relatively flat so that the length of the condensate flow path along the curved surface of the wall is equal to its component in the axial direction.

The component of the centrifugal acceleration in the condensate flow direction b_x is any function of the flow length ξ. A suitable function $b_x(\xi)$ shall now be determined such that the total resistance of the condensate film and wall is uniform over the entire surface of the drum, i.e.

$$\frac{w}{\lambda_w} + \frac{y}{\lambda_c} = \text{const.} \tag{6}$$

With the dimensionless wall thickness defined by

$$\delta = \frac{w}{L} \tag{7}$$

the total resistance can be expressed by an equivalent total wall thickness

$$\sigma = \delta + \phi . \frac{\lambda_w}{\lambda_c}. \tag{8}$$

Since the total resistance is to be constant along the surface of the drum we can write

$$\frac{d\sigma}{d\xi} = \frac{d\delta}{d\xi} + \frac{d\phi}{d\xi} . \frac{\lambda_w}{\lambda_c} = 0. \tag{9}$$

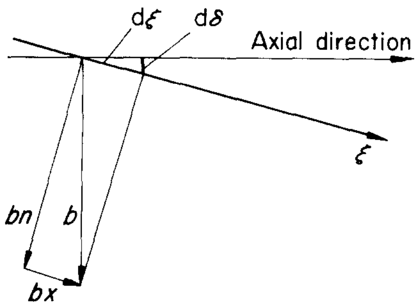

FIG. 1. Inclined inside surface in the acceleration field.

For small slopes, see Fig. 1, we can say

$$\frac{d\delta}{d\xi} = -\frac{b_x}{b_n}. \tag{10}$$

Combining (10) and (9) gives

$$\frac{d\phi}{d\xi} = \frac{\lambda_c}{\lambda_w} . \frac{b_x}{b_n}. \tag{11}$$

The function $b_x(\xi)$ must now be determined such that the film thickness ϕ satisfies (5) and (11). The differential equation (5) can not be solved for any function $b_x(\xi)$. Therefore we make an approximation which is verified later. From the results of [1] we can assume that the film thickness will be small compared to the length and that the film thickness is affected by the normal forces (represented by b_n) only very close to the end of the flow path, i.e. where $\xi \to 1$. Therefore we neglect the normal forces for this derivation taking

$$b_n = 0. \tag{12}$$

Introducing (12) into (5) yields (see also (44) and (45) of [1]) the local film thickness

$$\phi = \left(\frac{b_n}{2 . A}\right)^{\frac{1}{3}} . \left(\frac{\xi}{b_x}\right)^{\frac{1}{3}} \tag{13}$$

where b_n cancels against b_n in A. Differentiating (13) gives

$$\frac{d\phi}{d\xi} = \frac{1}{3} . \left(\frac{b_n}{2 . A}\right)^{\frac{1}{3}} . \xi^{-\frac{2}{3}} . b_x^{-\frac{1}{3}} . \left(1 - \frac{\xi}{b_x} . \frac{db_x}{d\xi}\right). \tag{14}$$

Substituting in (14) $d\phi/d\xi$ according to (11) and rearranging gives

$$\frac{b_x}{b_n} = \left(\frac{1}{3} . \frac{\lambda_w}{\lambda_c}\right)^{\frac{3}{4}} . \left(\frac{1}{2 . A}\right)^{\frac{1}{4}} . \left(1 - \frac{\xi}{b_x} . \frac{db_x}{d\xi}\right)^{\frac{3}{4}} . \xi^{-\frac{1}{2}}. \tag{15}$$

We postulate that the function $b_x(\xi)$ is of the form

$$b_x = K . \xi^a \tag{16}$$

and differentiating gives

$$\frac{\xi}{b_x} . \frac{db_x}{d\xi} = a. \tag{17}$$

For the special case treated in [1] $a = 1$, and (15) can be fulfilled for any value of $\xi > 0$, only if the ratio of wall thermal conductivity to condensate thermal conductivity is infinite, as was mentioned in the introduction. For all other cases where the ratio of conductivities is not infinite, (15) shows that the exponent, a, must be equal to $-\frac{1}{2}$ and therefore substituting for (17) in (15) gives

$$\frac{b_x}{b_n} = \frac{1}{2} \cdot \left(\frac{\lambda_w}{\lambda_c}\right)^{\frac{3}{4}} \cdot \left(\frac{1}{A}\right)^{\frac{1}{4}} . \xi^{-\frac{1}{2}} = \frac{1}{2} . G . \xi^{-\frac{1}{2}} \quad (18)$$

which defines G for the special case where $a = -\frac{1}{2}$ and the heat flux is uniform. Comparing (18) with (16) shows that $K = b_n . G/2$. Substituting for b_x/b_n in (10) according to (18) and integrating gives the dimensionless wall thickness

$$\delta = \delta_1 + G.(1-\xi^{\frac{1}{2}}) \quad (19)$$

where δ_1 is the dimensionless wall thickness at the end of the flow path (edge of groove, see Fig. 2) which can take any value within constructional constraints.

It can be seen from (19) that as ξ increases from 0 to 1, the wall thickness decreases proportionally to the square root of the length of the flow path.

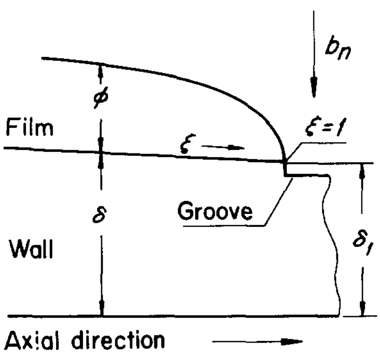

FIG. 2. Condensate film on the inside wall surface close to the edge of the condensate collecting groove. (See also Fig. 2 of [1].)

3. NUMERICAL SOLUTION OF EQUATIONS

With (16) and the right hand term of (18) the differential equation (5) can be written

$$\frac{d\phi}{d\xi} = G.\xi^a - \frac{1}{A} \cdot \frac{\xi}{\phi^3}. \quad (20)$$

For the special case treated in Section 2, G is defined by (18) and $a = -\frac{1}{2}$. Equation (20) was solved in [1] for the case in which $a = 1$, and for the limiting case in which $G = 0$ (and finite values of a). However, a general solution of (20) does not exist for all values of a and G. Therefore ϕ is determined by an incremental calculation.

Difficulties arise when ϕ is calculated incrementally as a function of ξ because at the starting point [boundary condition $\xi = 1$, $\phi = 0$, see [1], (12)] the slope $d\phi/d\xi$ is infinite. Solving for ξ as a function of ϕ is also difficult because there usually exists a maximum in ϕ. However these difficulties can be avoided by introducing the following substitutions

$$s = \phi^4 \quad (21)$$

$$t = \xi^2 \quad (22)$$

(20) now becomes

$$\frac{ds}{dt} = 2.G.t^{(a-1)/2}.s^{\frac{3}{4}} - \frac{2}{A}. \quad (23)$$

From the boundary condition $\xi = 1$, $\phi = 0$ [see [1], (12)], we get

$$t = 1 \rightarrow s = 0. \quad (24)$$

Replacing ds and dt in (23) by Δs and Δt in the usual manner gives in finite difference form

$$s_{i+1} = s_i + (t_{i+1} - t_i) . \left(2.G.t_i^{(a-1)/2}.s_i^{\frac{3}{4}} - \frac{2}{A}\right). \quad (25)$$

The incremental calculation begins at the end point given by the boundary condition in (24), and yields s as function of t and hence ϕ as function of ξ for any value of a and G.

In the special case where $a = -\frac{1}{2}$ another solution is possible. (23) yields ds/dt as a function only of s/t. Introducing s/t as an independent variable one can solve (23) by separating the variables in the usual manner. This gives t as a function of s/t, however, the function is a complicated definite integral which can be solved only incrementally and this special solution is more complicated than the general simple incremental method described above. Therefore we employ the general method also for the special case where $a = -\frac{1}{2}$.

4. RESULTS

Numerical results are first presented for the special case derived in Section 2. In the derivation the normal forces were neglected but in the incremental calculation they are taken into account.

Two cases are considered with different materials for the wall, the normal case of steel with $\lambda_w = 45$ W/Km [4] and the feasible case of aluminium with $\lambda_w = 206$ W/Km [4]. In both cases λ_c was taken as 0·689 W/Km [4].

The calculations were done for several values of the dimensionless group A and the results were similar. Tables 1 and 2 show the results obtained for $A = 10^{13}$ which was used also in [1]. The exponent is $a = -\frac{1}{2}$ and G was calculated according to (18). Of the many points used in the incremental calculations in which t was changed stepwise, only a few representative values are shown in the tables.

Table 1. Calculated values of wall profile, film thickness and overall resistance for steel wall with dimensionless groups $A = 10^{13}$ and $G = 0{\cdot}0129$

ξ	$(\delta-\delta_1)\times 10^4$ (19)	$\phi \times 10^4$ (25)	$(\sigma-\delta_1)\times 10^2$ (8)
0·0	129·22	0·0	1·2922
0·145	80·03	0·7549	1·2934
0·203	71·07	0·8924	1·2937
0·247	65·00	0·9855	1·2938
0·302	58·25	1·0892	1·2940
0·401	47·37	1·2562	1·2943
0·501	37·76	1·4037	1·2945
0·601	29·06	1·5372	1·2947
0·649	25·13	1·5974	1·2948
0·701	21·05	1·6600	1·2950
0·749	17·39	1·7163	1·2950
0·801	13·60	1·7744	1·2951
0·849	10·15	1·8274	1·2952
0·906	6·22	1·8877	1·2953
0·933	4·39	1·9158	1·2954
0·960	2·63	1·9426	1·2954
0·980	1·28	1·9563	1·2908
1·0	0·0	0·0	0·0

Table 2. Calculated values of wall profile, film thickness and overall resistance for aluminium wall with dimensionless groups $A = 10^{13}$ and $G = 0{\cdot}0404$

ξ	$(\delta-\delta_1)\times 10^4$ (19)	$\phi \times 10^4$ (25)	$(\sigma-\delta_1)\times 10^2$ (8)
0·0	404·34	0·0	4·0434
0·144	250·97	0·5132	4·0443
0·202	222·73	0·6078	4·0444
0·246	203·64	0·6716	4·0445
0·301	182·44	0·7426	4·0447
0·414	144·33	0·8701	4·0449
0·501	118·14	0·9578	4·0450
0·601	90·92	1·0488	4·0452
0·649	76·64	1·0899	4·0452
0·701	65·87	1·1327	4·0453
0·749	54·41	1·1710	4·0454
0·801	42·55	1·2107	4·0454
0·849	31·75	1·2468	4·0454
0·906	19·45	1·2880	4·0456
0·933	13·72	1·3072	4·0456
0·960	8·23	1·3255	4·0456
0·980	4·00	1·3397	4·0456
1·0	0·0	0·0	0·0

The results clearly illustrate that although the wall thickness $\delta-\delta_1$ and the film thickness ϕ change drastically in the flow direction the overall resistance, represented by the dimensionless equivalent total wall thickness $\sigma-\sigma_1$, is constant over virtually all of the flow path (see Fig. 3).

The overall resistance changes at the two ends of the flow path, i.e. at $\xi\to 0$ and $\xi\to 1$. At the point $\xi\to 1$ the overall resistance becomes zero because of the effect of the normal forces b_n. However these forces do not affect the film along the rest of the flow path thus confirming the assumption used in Section 2 of neglecting b_n. At the point $\xi\to 0$, the change from uniformity is due to the slope of the wall which becomes $-\infty$ and the assumption of (10) is not valid. Also at this point the heat flow is not normal to the surface of the drum as was assumed in the derivation. These two effects at $\xi\to 0$ have little effect on the film downstream of this point, as is illustrated by the results. This was also verified by a separate calculation with which these effects could be estimated.

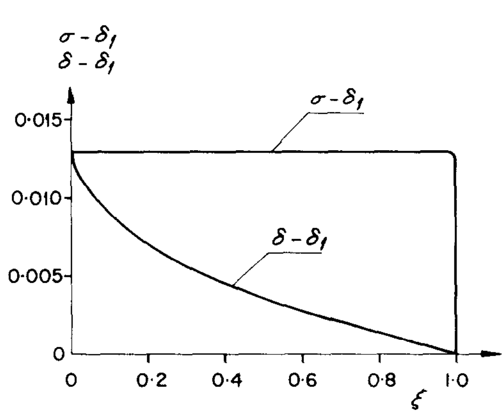

FIG. 3. Wall profile and total resistance for $a = -\frac{1}{2}$, $A = 10^{13}$ and $G = 0{\cdot}0129$ (steel).

The profile of the wall was calculated for a particular value of A which is determined by normal operating conditions, heat flux and drum speed. In practice however these operating conditions might change, e.g. the drum might be operated at a speed different from the one used in the design of the wall profile. The effect, on the uniformity of heat flux, of operating at an incorrect speed was investigated by another set of calculations. The value of G in (19) and (25) was calculated with the design value $A = 10^{13}$ while the value of A actually used in (25) was changed by altering the value of ω in (4). For a given wall profile the value of G is independent of speed since the ratio b_x/b_n is independent of b_n and is merely a function of the slope of the wall.

Tables 3 and 4 illustrate the effect of a change in speed of +10 and −10 per cent for two different wall materials, steel and aluminium. The wall thickness δ is the same as in Tables 1 and 2. It can be seen that the total resistance, represented by $\sigma-\sigma_1$, changes only very slightly along the flow path with this variation in speed. In an actual drying drum the dimensionless minimum wall thickness δ_1 may assume values of 0·01–0·02. Thus the variation in the total equivalent wall thickness, σ, will be less than the variation of $\sigma-\delta_1$ shown in the final column of Tables 3 and 4.

Table 3. Calculated overall resistance for different operating speeds (± 10 per cent of design speed)

ξ	$(\sigma-\delta_1)\times 10^2$ (8)		
	$\frac{\omega}{\omega_d}=0{\cdot}9$	$\frac{\omega}{\omega_d}=1{\cdot}0$	$\frac{\omega}{\omega_d}=1{\cdot}1$
0·0	1·292	1·292	1·292
0·145	1·329	1·293	1·263
0·203	1·336	1·294	1·258
0·247	1·341	1·294	1·254
0·302	1·346	1·294	1·250
0·401	1·354	1·294	1·244
0·501	1·361	1·295	1·238
0·601	1·368	1·295	1·233
0·649	1·371	1·295	1·230
0·701	1·374	1·295	1·228
0·749	1·377	1·295	1·226
0·801	1·380	1·295	1·224
0·849	1·382	1·295	1·222
0·906	1·385	1·295	1·219
0·933	1·387	1·295	1·218
0·960	1·388	1·295	1·217
0·980	1·382	1·291	1·213
1·0	0·0	0·0	0·0

Table 4. Calculated overall resistance for different operating speeds (± 10 per cent of design speed)

ξ	$(\sigma-\delta_1)\times 10^2$ (8)		
	$\frac{\omega}{\omega_d}=0{\cdot}9$	$\frac{\omega}{\omega_d}=1{\cdot}0$	$\frac{\omega}{\omega_d}=1{\cdot}1$
0·0	4·043	4·043	4·043
0·144	4·156	4·044	3·950
0·202	4·177	4·044	3·932
0·246	4·191	4·045	3·921
0·301	4·206	4·045	3·908
0·414	4·234	4·045	3·885
0·501	4·254	4·045	3·869
0·601	4·273	4·045	3·852
0·649	4·283	4·045	3·844
0·701	4·292	4·045	3·837
0·749	4·300	4·045	3·830
0·801	4·309	4·045	3·822
0·849	4·317	4·045	3·816
0·906	4·326	4·046	3·808
0·933	4·330	4·046	3·805
0·960	4·334	4·046	3·802
0·980	4·337	4·046	3·800
1·0	0·0	0·0	0·0

This demonstrates that changes in speed of ± 10 per cent have little effect on the uniformity of heat flux. Further calculations have shown that even higher deviations from the design speed can be tolerated. The optimum speed and the allowable tolerance, for a given wall profile (value of G), however, should be determined experimentally.

In all cases calculated, where $a = -\frac{1}{2}$ and G was estimated from (18) the mean heat-transfer resistance was lower or more uniform than the case considered in [1], when operating under similar conditions, i.e. the same value for A.

The incremental method (25) was used to calculate many other cases with various values of a and G. In particular the case of a cone with a constant slope ($a = 0$), which is easier to manufacture was investigated. However, a generally valid satisfactory uniformity of heat flux was not obtained. As a typical example Fig. 4 gives the total dimensionless resistance σ for the values $a = 0$, $G = 0{\cdot}02$, and $A = 10^{13}$.

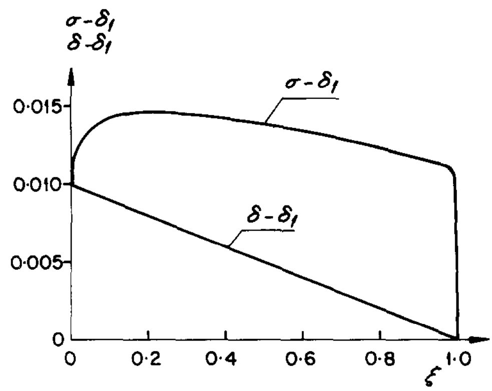

FIG. 4. Wall profile and total resistance for $a = 0$, $A = 10^{13}$ and $G = 0{\cdot}02$ (steel).

During the incremental solution of (25) the film thickness was integrated and a mean value determined to give the reciprocal of the Nusselt number. It was found that the Nu, calculated for a variety of different values of A, a and G were in good agreement with those obtained from (35) and (48) in [1] i.e.

$$Nu = \left[\left(\frac{4-a}{3}\right)^{\frac{31}{7}} . (G.A)^{\frac{31}{21}} + (0{\cdot}86 . A)^{\frac{31}{28}}\right]^{\frac{7}{31}} . \qquad (26)$$

For the special case developed in Section 2 with $a = -\frac{1}{2}$ and G defined by (18) we can simplify (26)

$$Nu = A^{\frac{1}{4}} . \left[\left(5{\cdot}06 . \frac{\lambda_w}{\lambda_c}\right)^{\frac{31}{28}} + 0{\cdot}84\right]^{\frac{7}{31}} \qquad (27)$$

and for normal ratios of λ_w/λ_c (27) approaches closely to the limiting case where normal forces, b_n are neglected (see also [1]).

REFERENCES

1. W. Roetzel, Improving heat transfer in steam-heated fast rotating paper drying drums, *Int. J. Heat Mass Transfer* **18**(1), 79–86 (1975).
2. S. T. Han, Drying of paper, *Tappi* **53**(6), 1034–1046 (1970).
3. W. Nusselt, Die Oberflächenkondensation des Wasserdampfes, *Z. Ver. D. Ing.* **60**, 541–546, 569–575 (1916).
4. D. Q. Kern, *Process Heat Transfer*, p. 799. McGraw-Hill/Kōgakusha, Tokyo (1950).